UNBALANCED BIDDING MODELS WITH RISK CONSIDERED

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So-called "Unit Price Proposals" indicate the sponsor's anticipated items, and the estimated number of each item deemed necessary to achieve the objective of the proposal. Bids are solicited per unit of each item from which the total bid is extended. In general, the contract is awarded on the basis of the total bid, whereas the payments are made on the basis of the amounts actually materialized and on their corresponding unit bids.

An unbalanced bid is one which does not necessarily reflect the bidder's economic price to provide a unit of the corresponding item. There are several motivations for an unbalanced bidding strategy. First, by manipulating the unit bids, the bidder effectively, controls the timing of the reimbursements thereby alleviating much of the financing burden of the proposal. Second, in general, the solicited amounts do not necessarily reflect the best estimates of the amounts required. The bidder may conduct an investigation to accurately assess the required quantities. Thus, if the bids are held lower on items for which the requirements are overestimated, the bidder, without increasing his total bid, may increase his expected returns by bidding high on those items which are underestimated.

To optimally unbalance a bid Stark (1968) proposes a standard linear programming (LP) scheme. He maximizes expected returns subject to constraints which require (1) that the total bid does not exceed the one which is previously determined through game theoric considerations,(1) (2) that the individual bids do not violate certain techonological and formality relationships (e.g., the bid for a certain item must not be greater than that of another), and (3) that the rate at which payments are made by the sponsor for completed items be in line with a comfortable financial planning of the proposal.

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(1) Clearly determination of the total bid and optimally unbalancing the total bid are two separate problems.

The main objection to this formulation stems from its exclusion of a very genuine concern of the bidder. By manipulating the unit bids, the bidder assumes some risk in terms of financial losses which is completely ignored by the LP formulation. A different development that takes explicit account of the risk involved is the subject of this paper.

Proposed Models

We present three formulations which differ slightly in terms of conceptualization, but largely in terms of solution procedures. The main idea, common to all, is to treat whatever measure of risk used as a parameter to obtain optimum exepcted returns as a function of risk. We will refer to this relationship as the "efficient locus." A utility function similar to those used in portfolio selection analysis will be used together with the efficient locus to single out the most desirable solution on the efficient locus (for further discussion of these points see Markowitz (1959)).

Notation.

x_i	:	unit bid for item i.
y_i	:	out-of-pocket cost per unit of item i.
N_i	:	a chance variable describing the actual requirements for item i.
$\bar{N}_i$	:	amount solicited.
$f_i(N_i)$	:	density function of N_i.
μ_i	:	mean of N_i
σ^2_i	:	variance N_i
n	:	number of items in the proposal.
D	:	Total Bid.

Problem I (P. I)

The first formulation is a chance constrained problem which maximizes the expected value of the returns subject to i) total bid constraint (Eqı. 2); ii) formality constraint which require that unit bids for certain items cannot exceed that of certain others, and limit the bids to given intervals $[L_i, U_i]$, (Eqs. 3 and 4); and iii) probability constraint which requires that the probability of attaining a minimum return is at least as large as α.

$$\text{Max } z = E[\sum_i (x_i - y_i) N_i] \tag{1}$$

$$\text{s. t.} \quad \sum_i x_i \bar{N}_i = D \tag{2}$$

$$x_r - x_e \leqslant 0 \qquad \text{for some r and e} \tag{3}$$

$$L_i \leqslant x_i \leqslant U_i \qquad \text{for all i} \tag{4}$$

$$\Pr [\sum_i (x_i - y_i) N_i \geq T] > \alpha. \tag{5}$$

To solve P. I., the last constraint must be put in some operational form. This would require that we make some assumptions on the shapes of f_i's. If f_i's are normal then $\sum_i (x_i - y_i) N_i$ will also be normal; if normality of f_i's is not justified, with N_i independent and n sufficiently large $\sum_i (x_i - y_i) N_i$ will approach normality as a consequence of the central limit theorem. The parameters of the returns distribution are

$$\mu_r = \sum_i (x_i - y_i) \mu_i \quad \text{and} \quad \sigma^2_r = \sum_i \sum_j (x_i - y_i) (x_j - y_j) \sigma^2_{ij},$$

where $\sigma^2_{ij} = E[(N_i - \mu_i)(N_j - \mu_j)] = 0 \quad i \neq j$

Constraint (5) can now be replaced with

$$T - \sum_i (x_i - y_i) \mu_i \leqslant Z\alpha [\sum_i \sum_j (x_i - y_i) (x_j - y_j) \sigma^2$$

Technically, P.I. is a non-linear, non-separable problem for which efficient solution schemes are not available. One of the several gradient search techniques may be employed to solve the problem. This would, however, be very impractical due to: (1) the inherent computational burden of the search techniques, and (2) treating α and T as parameters would require many solutioins to P.I. for various combinations of α and T.

Problem II (P. II)

The following formulation uses the variance of the returns function as the measure of the risk; this is a well accepted measure in the portfolio selection problem which is conceptually similar to the problem at hand[2]. Specifically we want to minimize the functional,

$$\sum_i \sum_j (x_i - y_i) (x_j - y_j) \sigma^2_{ij}$$

(2) For instance see Markowitz (1959).

s.t. Eqs. (2), (3), (4) and

$$\sum_i (x_i - y_i)\,\mu_i \geq T$$

The reader should note that minimizing the variance for different levels of expected returns will define the same efficient locus as maximizing the expected returns for various levels of variance. Posing the problem in this fashion is computationally advantageous for P. II is a quadratic programming problem (optimizing a quadratic form subject to linear restrictions) for which solution schemes are available, such as the quadratic simplex algorithm.

Problem III (P. III)

Here we present a simplified version of the problem which will be used as a vehicle to establish several important results pertaining to the behavior of the efficient locus. Assuming Cov $(N_i, N_j) = 0$, and disregarding restrictions (3) and (4) we have

$$\max z = \sum_i t_i\,\mu_i \qquad (9)$$

$$\sum_i t_i\,\bar{N}_i = B \qquad (10)$$

$$\sum_i t_i^2\,\sigma_i^2 \leqslant \sigma^2 \qquad (11)$$

where σ^2 is the desired variance of the total returns, $t_i = x_i - y_i$ and $B = D - \sum_i y_i\,\bar{N}_i$.

It is easy to show that restrictions (10) and (11) are convex sets therefore the feasible region of P. III is convex. It is a well known fact that a linear function achieves its extrema over a convex set at the boundary of the feasible set. Hence, to solve P. III it is sufficient to examine the stationary points of the lagrangian:

$$L(t,\lambda_1,\lambda_2) = \sum_i t_i\mu_i - \lambda_1 \left(\sum_i t_i\,\bar{N}_i - B\right) - \lambda_2\left(\sum_i t_i^2\sigma_i^2 - \sigma^2\right) \qquad (12)$$

where $t = (t_1, t_2, \ldots\ldots t_n)$.

The stationary points of L(.) satisfy the following conditions:

$$\frac{\partial L}{\partial t_i} = \mu_i - \lambda_1\bar{N}_i - 2\lambda_2 t_i\sigma_i^2 = 0 \qquad i=1,\ldots,n \qquad (13)$$

$$\frac{\partial L}{\partial \lambda_1} = \sum_i t_i \bar{N}_i - B = 0$$

$$\frac{\partial L}{\partial \lambda_2} = \sum_i t^2_i \sigma^2_i - \sigma^2 = 0 \qquad (15)$$

Above conditions with $\lambda_1, \lambda_2 \geq 0$ are necessary and sufficient for a global optimum due to the convexity of the feasible region and the linearity of the objective function. The solutions to the above system can be obtained by first solving the Eqs. (13) for t_i and substituting in (14) and (15) to yield two equations in two unknowns, λ_1, λ_2. Since (15) is quadratic, in general, there will be two pairs (λ'_1, λ'_2), $(\lambda_1'', \lambda_2'')$ satisfying the system.

$$\lambda'_1 = \left[MN + \left(\frac{MN^2 - NN \cdot MM}{1 - \frac{\sigma^2}{B^2} NN} \right)^{1/2} \right] / NN \qquad (16)$$

$$\lambda'_2 = (MN - NN \cdot \lambda'_1) / 2B \qquad (17)$$

$$\lambda''_1 = \left[MN - \left(\frac{MN^2 - NN \cdot MM}{1 - \frac{\sigma^2}{B^2} NN} \right)^{1/2} \right] / NN \qquad (18)$$

$$\lambda''_2 = (MN - NN \cdot \lambda''_1) / 2B \qquad (19)$$

where

$$MN = \sum_i \frac{\mu_i \bar{N}_i}{\sigma_i^2} \geq 0, \quad MM = \sum_i \frac{\mu^2_i}{\sigma_i^2} \geq 0 \quad \text{and } NN = \sum_i \frac{\bar{N}_i^2}{\sigma_i^2} > 0.$$

Solutions to P. III

In this section we establish the optimality conditions of P .III. The following results will be used in the development that follows.

Lemma 1 $MN^2 - MM.NN \leqslant 0$ if $N_i \geq 0$, $\mu_i \geq 0$ $i=1,2...,n$.

proof: let $m_i = \frac{\mu_i}{\sigma_i}$, $n_i = \frac{\bar{N}_i}{\sigma_i}$ we have

$$(\sum_i m_i n_i)^2 - (\sum_i m^2)(\sum_i n^2_i) \leqslant 0$$

which reduces to a sum of differences squared as follows :

$$-\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (m_i n_j - m_j n_i)^2 \leqslant 0.$$

Lemma 2a For $B>0$, $\lambda_1^* \neq \lambda_1'$ (Eq. 16)

where λ^*_i denote the optimal multipliers.

proof: assume $\lambda_1^* = \lambda'_1$ then Eq. (17) implies

$$\lambda_2 = - K/2B$$

where K is the positive square root of $\dfrac{MN^2 - NN.MM}{1 - \sigma^2.NN/B^2}$

Since by assumptions $B>0$ and $K>0$ $\lambda'_2<0$, thus λ'_1 cannot be optimum.

Lemma 2b For $B<0$, $\lambda^*_1 \neq \lambda''_1$ (Eq. 18)

proof: similar to Lemma 2a.

Lemma 3 K exists if $\sigma^2>B^2/NN$

proof: by Lemma 1 $MN^2 - MM.NN \leq 0$; for the inside of the radicand to be positive we must have $1 - \dfrac{\sigma^2}{B^2} NN < 0$ which implies $\sigma^2>B^2/NN$.

Lemma 4 $B^2/NN < B^2 . MM/MN^2$

proof: directly follows from Lemma 1.

Theorem 1 (Main Theorem)

i) For $B>0$ and $\sigma^2>B^2\ MM/MN^2$ the unique optimum multiplier values for P. III are:

$$\lambda^*_1 = \lambda''_1 = (MN - K) / NN$$

$$\lambda^*_2 = \lambda''_2 = K / 2B.$$

ii) For $B<0$ and $\sigma^2>B^2/NN$ the unique optimum multiplier values to P. III are:

$$\lambda^*_1 = \lambda'_1 = (MN+K)\ NN$$

$$\lambda^*_2 = \lambda'_2 = - K / 2B$$

proof: i) From Lemmas 3 and 4, K exists, thus (MN-K)/NN exists. Since $NN>0$, $MN—K\geq 0$ is sufficient for $(MN—K)/NN\geq 0$. $MN—K\geq 0$, in turn, requires that $\sigma^2>B^2.MM/MN^2$. Moreover, K/2B exists and it is positive.

ii) Since $MN\geq 0$, and $NN>0$, existence of K is also sufficient for $(MN+K)/NN\geq 0$. By lemma 3 K exists if $\sigma^2>B^2/NN$. Also since $B<0$, —K/2B exists and it is positive.

Efficient Locus

Solving P. III for different feasible values of σ^2 a relationship between z and σ^2 is obtained. The curve of this relationship will be called the **efficient locus.** We next consider the behavior of the efficient locus.

Theorem 2 Efficient locus is concave in feasible σ^2's.

Proof: $\dfrac{dz}{d\sigma^2} = \lambda^*_2$ by the property of the multipliers.

$$\lambda^*_2 = \begin{cases} \dfrac{K}{2B} & \text{if } B > 0 \\ \dfrac{—K}{2B} & \text{if } B < 0 \text{ by lemma 2a and 2b.} \end{cases}$$

$$\frac{d^2z}{d(\sigma^2)^2} = \begin{cases} \dfrac{NN}{8B} \cdot \dfrac{(MN^2\ MM.NN)}{(B^2 — \sigma^2.NN)^3} & \text{if } B > 0 \\ — \dfrac{NN}{8B} \cdot \dfrac{(MN^2\ MM.NN)}{(B^2 — \sigma^2.NN)^3} & \text{if } B < 0 \end{cases}$$

Second derivative of expected returns with respect to σ^2 is positive under both conditions, thus the efficient locus is concave.

In Fig. 1 the shaded area represents the feasible solutions to P. III. Those points that lie on the upper half of the boundary correspond to $\lambda_2\geq 0$. therefore they are global optima, where as the lower half of the boundary corresponds to solutions for which $\lambda_2\leq 0$.

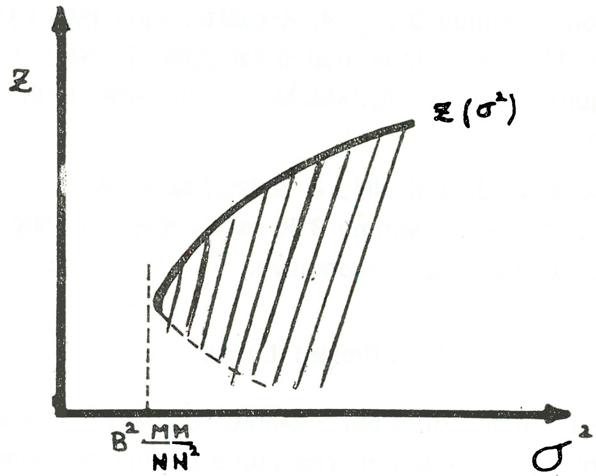

Figure 1. The Efficient Locus when $B \geq 0$.

Optimum Decisions

We remind the reader that the efficient iocus is the collection of the optimum solutions to problem III for different levels of risk. The sahpe of the efficient locus clearly establishes the trade off between expected returns and the corresponding risk level. On the basis of this, the logical question becomes determining which one of those points on the efficient locus is the most desirable one. One way of resolving this dilemma is to resort to some utility considerations.

It is customary, especially in investment analysis, to postulate a utility function of the form $U = f(z, \sigma^2)$ with $\frac{\partial U}{\partial z} > 0$ and $\frac{\partial U}{\sigma^2} < 0$. These inequalities are justified on the basis of psychological assumptions about human behavior. The reader is referred to Farrar (1962) for extensive discussions of this type of utility funcitons. $\frac{\partial U}{\partial z} > 0$ and $\frac{\partial U}{\sigma^2} < 0$ imply iso-utility contours with positive slopes. In other words, increased risk must be accompanied by increased expected returns to preserve the same level of utility. Although much less general agreement is available on the curvature of the iso-utility countours, economists usually tend to draw them convex, suggesting that the decision maker's aversion to further risk is an increasing function of the amount already being taken (See Farrar (1962)). The tangency point of iso-utility contours and the efficient locus determines the most desirable solution like in Fig. 2.

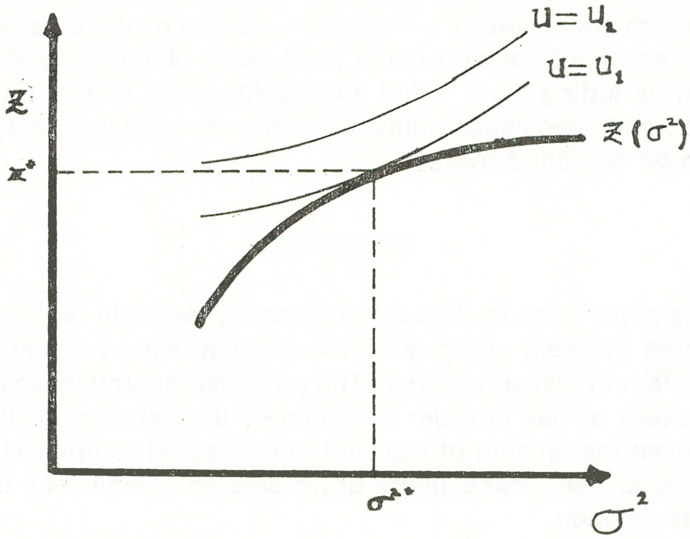

Figure 2. Interaction of z (σ^2) and U (z, σ^2).

Example

μ_i	$\bar{N}_i$	σ^2_i
15	20	16
20	20	9
30	20	4

B=10; $U=z-\sigma^2$

By solving P. III for various of σ^2 following points on the efficient locus are obtained.

z	σ^2	$z - \sigma^2$
18.65	3.00	15.65
20.79	5.00	15.79 optimum (3)
22.48	7.00	15.48
23.91	9.00	14.91
25.19	11.00	14.19
26.35	13.00	13.35

The corresponding optimum solution is: $t_1 = -.27$; $t_2 = -.18$; $t_3 = .94$. The unit bids are easily obtained from the identities $t_i = x_i - y_i$.

(2) This may not be the exact optimum since σ^2 is varied in discrete intervals.

It should be clear that for a linear utility function it is not at all necessary to solve the problem repeatedly, for the optimum solution since optimality occurs at the point where the slope of the utility contour, the coefficient of σ^2, equals the slope of the efficient locus, λ_2. A more exact solution could have been obtained to the above problem by letting $\lambda_2 = 1$ and solving for t_i's,λ_1 and σ^2. In general however, with non-linear utility functions the efficient locus would have to be searched over σ^2.

SUMMARY

This paper tried to deal with risk encountered in bidding situations. Three different versions of the problem were presented, each with a different measure of risk. The proposed approach was to treat the measure of risk in order to establish the behavior of the trade off between the amount of risk and the expected returns. The paper also tried to use some utility arguments to single out the most desirable solution.

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ÖZET

Bazı İhalelerde Optimal Teklif Stratejileri

Bu çalışma belirli bazı ihale durumlarında optimal teklif stratejilerini araştırmaktadır. Bu tür stratejilerin uygulanmasında karşılaşılması zorunlu olan rizikonun açık bir biçimde göz önüne alınması çalışmanın asıl amacını oluşturmaktadır. Problemin üç ayrı formulasyonu sunulmuş, bunlardan bir tanesi derinlemesine araştırılarak riziko ile beklenen optimum kazanç arasındaki ilişki saptanmıştır. Ayrıca, optimum kazanç-riziko seviyelerinin fayda fonksiyonları aracılığı ile elde edilebileceği ileri sürülmüştür.